will be published in Ann. d'Astroph. Vol. 29 #4,

A THEORY OF THE ORIGIN AND EVOLUTION OF CONTACT BINARIES (*)

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Abstract. — Following Schatzman (1962) we propose that the orbital angular momentum of binaries may be dissipated through mass ejection along magnetic lines of force. It brings the separation of two component stars closer and closer, such that in some cases contact binaries, like W UMa systems, may be formed in this way.

If the dissipation of angular momentum continues after the two components come into direct contact, the course open for the binary is to transfer mass from the less massive to the more massive component. Three observational results — (1) the mass ratios of the W UMa systems, (2) the negative correlation between the axial rotation and the frequency occurrence of spectroscopic binaries in different clusters and associations, and (3) the stars of hydrogen-poor and helium-rich atmospheres — are discussed in the light of this suggestion.

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The contact binaries, like W UMa stars, have been found abundant in the galactic system (Shapley 1948). However, their formation remains a great mystery, because the two components simply could not have formed

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(*) A part of this investigation was carried out when the author was associated with the Department of Space Science and Applied Physics, Catholic University of America, Washington, D.C., and Goddard Space Flight Center, National Aeronautics and Space Administration, Greenbelt, Maryland.

so close together. Otherwise, the two stars would have engulfed each other during the pre-main-sequence stage of evolution. It is then difficult to envisage a separation of two gaseous spheres from a single one.

Many theories for the origin of close binaries in general have been proposed (e.g., Hynek 1951). For the contact binaries of W UMa types we face three possibilities: (1) fission from a single rapidly rotating star, (2) contraction of the orbit in a resisting medium, and (3) evolution from other close binaries. Most astronomers, including Jeans (1944), have ruled out the fission theory in its original form. Recently, Roxburgh (1965) has revived it. But this revived version does not remove all the difficulties encountered by the fission theory.

The idea of a resisting medium which dissipates the dynamical energy of a binary system and thereby reduces its separation faces the fact that in the interstellar medium the densities are not high enough to do the required work. Regarding the third possibility Struve (1950) has suggested that the contact binaries of the W UMa type are the product of evolution of more massive contact binaries such as U Coronae Borealis which is supposed to owe its existence again to fission of rapidly rotating stars. Hence, Struve's suggestion does not go beyond the fission origins for the contact binaries. In view of the difficulties encountered by the two conventional theories, we shall present here a theory for the formation of contact binaries based on a new mechanism of angular momentum dissipation (Schatzman 1962) whose importance is only recently realized.

There is increasing evidence that intense magnetic activities prevail in the early stage of stellar evolution. Several empirical results which are otherwise unexplainable can now be understood in terms of electromagnetic interaction taking place on the stellar surface. These theories have been recently summarized (Huang 1965b). In particular we should mention the overabundance of lithium in T Tauri stars as compared with its proportion in the solar atmosphere (Bonsack and Greenstein 1960; Herbig 1962). According to Fowler, Greenstein and Hoyle (1962), this anomaly arises from the spallation process that proceeds on the surface of these stars; the high-energy particles, dominantly protons, that are responsible for spallation are supposed to be accelerated by the same electromagnetic force that produces other phenomena, such as flares (Poveda 1964), ejection of matter (Herbig 1957), etc. Finally, Wilson (1963) has found a probable correlation between chromospheric activity (as seen from the H-K emission) and age in main-sequence stars in the sense that the activity decreases with age. Following what has been found in the solar chromosphere (Babcock and Babcock 1958; Osterbrock 1961), he has also suggested that magnetic field strength over the stellar surface may determine the strength of H-K emission. If so, there must be a strong field in the early phase of stellar evolution, agreeing with the conclusion obtained from other considerations, as we have already seen.

Ionized particles ejected along the magnetic lines of force that rotate with the star acquire a large amount of angular momentum. When these particles are lost to the system or are absorbed by the surrounding medium, the angular momentum they carry is lost to the star. The loss of angular momentum in this way provided an effective means for braking stellar rotation (Schatzman 1962). Indeed, we have found that the statistical behavior of stellar rotation seems to agree with the concept of braking (Huang 1965a). Now if the spin angular momentum of the star can be dissipated this way, it is equally likely that the orbital angular momentum of a binary system may be similarly dissipated through electromagnetic interaction.

The dissipation of the orbital angular momentum may take two slightly different idealized forms. If the magnetic lines of force of two component stars are connected, the orbital angular momentum will be directly transported out by the ejected particles along the lines of force. If, on the other hand, the two component stars are not magnetically linked, it is the spin angular momentum that is directly dissipated by the ejected matter. However, since the two components of a close binary are always coupled dynamically even if not magnetically, the loss of the spin angular momentum eventually will lead to the decrease of the orbital angular momentum. In reality, with magnetic activities occurring on the surfaces of both components, the two stars are likely coupled by magnetic lines because any instability at zero point can easily connect the magnetic lines emanating from two stars. Here we see a means to bring the two components of the binary together more effectively than a resisting medium. In fact this appears to be the only reasonable way that contact binaries that abound in the solar neighborhood can be formed. In the following section we will develop a preliminary theory for the origin of contact binaries based on this idea.

2. Formation of Contact Binaries

Consider a binary system whose two components are revolving around each other in circular orbits for the sake of simplicity. Let the separation between the two components be a. Hence, if we donote by M_i, R_i, $R_i k_i$ (i = 1, 2) respectively, the masses, radii, and radii of gyration of the two components, the total angular momentum of the system, $\mathfrak{M}$, becomes

$$\mathfrak{M} = \sum_{i=1}^{2} I_i \omega_i + \mu a^2 \omega \tag{1}$$

where

$$\mu = \frac{M_1 M_2}{M_1 + M_2}, \qquad I_i = M_i R_i^2 k_i^2 \quad (i = 1, 2) \tag{2}$$

while ω_1 and ω_2 are respectively spin angular velocities of the two components and ω angular velocity of orbital revolution given by

$$\omega = \left[\frac{G(M_1 + M_2)}{a^3} \right]^{1/2} \tag{3}$$

with G as the gravitational constant.

We shall study separately three idealized cases of binaries whose angular momentum is being steadily dissipated. (1) The radii R_1 and R_2 of the two components change with time according to gravitational contraction but the two components are so far apart that the orbital motion and axial rotation are coupled magnetically, though perhaps not dynamically. This case perhaps applies to the early stage of drifting together of two components in a binary of a fairly large separation. (2) The radii R_1 and R_2 become constant. This would be the case when the two components have reached the main sequence and their separation has become quite close. (3) The two components are already in physical contact. The first two cases will be discussed in this section, leaving the third one for the next section.

<u>Case 1</u>. Let us first consider the variation of R_1 and R_2. According to Hayashi's (1961) theory of evolution for pre-main-sequence stars, the evolutionary track on the H-R diagram is dominantly vertical and their internal structure is based on convective equilibrium. This is especially true for the late stars. Hence, as a simplification, we shall assume the effective temperature, T_i, of the component to be constant in the entire course of evolution towards the main sequence. During its contracting stage, the luminosity of a star is supplied by its gravitational energy,

$$\Omega_i = -\alpha \frac{G M_i^2}{R_i} \qquad (i=1,2) \tag{4}$$

where α is a constant equal to 6/7 as long as the star remains in the state of convective equilibrium. We now assume that although it carries away a large amount of angular momentum, the mass ejected from the star is negligible. Therefore, in the following treatment we will take M_i ($i = 1, 2$) as constant in the course of evolution. It follows that the change of gravitational energy is only through a change in the radius. From the virial theorem we have

$$\frac{1}{2}\alpha\frac{GM_i^2}{R_i^2}\frac{dR_i}{dt} = -L_i, \qquad (i=1,2) \tag{5}$$

where the ratio of specific heats have been set equal to 5/3 and the luminosity of each component, L_i, is given by

$$L_i = 4\pi R_i^2 \sigma T_i^4, \tag{6}$$

σ being the Stefan-Boltzmann constant. Since the effective temperature, T_i ($i = 1, 2$) is assumed to be constant during evolution, equations (5) and (6) then yield

$$\frac{1}{R_i^3} = \frac{1}{R_{i,0}^3}\left(\frac{6t}{\lambda_i} + 1\right), \qquad (i=1,2) \tag{7}$$

where

$$\lambda_i = \frac{\alpha G M_i^2}{R_{i,0}}\frac{1}{4\pi R_{i,0}^2 \sigma T_i^4} \qquad (i=1,2) \tag{8}$$

is a time scale related to gravitational contraction and $R_{i,o}$ is the value of R_i at $t = 0$.

Next we consider the rate of angular momentum dissipation, which must be directly proportional to the rate of mass ejection. The latter likely increases with the luminosity because it is perhaps ultimately due to the convective energy flow that activates the mass ejection. If we denote m the mass ejected by the two components, the rate of mass ejection may be written as

$$\frac{dm}{dt} = \beta \sum_{i=1}^{2} \left(\frac{L_i}{L_{i,f}} \right)^n \tag{9}$$

where β is a parameter with the dimension of mass over time and $L_{i,f}$ represents the luminosity of each component (i = 1, 2) at the final stage of contraction (to be identified with the main sequence as an approximation). Similarly, we will denote $R_{i,f}$ as the final radius of the i component. Also, the rate of loss of angular momentum must be proportional to the angular velocity of the star, because the angular momentum carried away by ejected mass along the magnetic lines of force that rotate with the binary is directly proportional to the orbital angular velocity. Hence, we may write

$$\frac{d\mathfrak{M}}{dt} = -\beta \ell^2 \sum_{i=1}^{2} \left(\frac{L_i}{L_{i,f}} \right)^n \omega . \tag{10}$$

In writing this way, ℓ may be taken as the average effective radius (from the center of mass of the binary system) of points at which charged particles are decoupled from the magnetic lines of force. One of possible decoupling processes occurs when the charged particles enter into a cool medium of little ionization.

Since a is large in this case, we may neglect the first term on the right side of equation (1) in order to see roughly the manner the two component stars drift together. Substitution of equation (1) into equation (10) yields a differential equation which may be integrated to derive the following result:

$$1-x^2 = \frac{2}{3-2n}\frac{1}{\tau_1}\sum_{i=1}^{2}\lambda_i\left(\frac{R_{i,c}}{R_{i,f}}\right)^{2n}\left[\left(\frac{6t}{\lambda_i}+1\right)^{(3-2n)/3}-1\right] \tag{11}$$

where

$$x = \frac{a}{a_0} \quad \text{and} \quad \tau_1 = \frac{\mu q_0^2}{\beta l^2} \tag{12}$$

a_0 being the initial value of a.

We can easily integrate equation (9) in the same way as equation (10) and ~~indeed~~ obtain the same summation as that appearing in equation (11). Combining the results, we derive a simple relation

$$1-x^2 = \frac{4ml^2}{\mu q_0^2} \tag{13}$$

which is independent of n.

Equation (11) describes in general how the two components approach each other as a result of angular momentum dissipation and is valid before either component reaches the main sequence, i.e., for t less than the contracting time scale $t_{c,1}$ which may be obtained by setting R_1 equal to its main-sequence value, $R_{1,f}$, in equation (7).

Let us consider a special case of $M_1=M_2$ and $n = 1$. Write $\lambda_1=\lambda_2=\lambda$ $t_{c1}=t_{c2}=t_c$, $R_1=R_2=R$. If we denote x_f as the value of x when both components have reached the main sequence, it can easily be derived from equation (11) that

$$x^2 = 1-(1-x_f^2)\,\varphi(t;r) \tag{14}$$

where

$$\varphi(t;r) = \frac{1}{r-1}\left\{\left[(r^3-1)\frac{t}{t_c}+1\right]^{1/3}-1\right\} \qquad 0\le t\le t_c \tag{15}$$

and

$$r = \frac{R_0}{R_f} \tag{16}$$

is the contraction factor of the radius. Equation (14) gives the variation with time of separation from $x = 1$ to $x = x_f$ for different values of r. Hence, r in $\varphi(t;r)$ affects only slightly the manner in which x decreases from 1 to x_f. The actual amount of decrease in separation is determined by x_f which is given by

$$1-x_f^2 = \frac{12r^2}{r^2+r+1}\,\frac{t_c}{\tau_1} \tag{17}$$

If $\mathcal{M}_0$ denotes the orbital angular momentum at $t = 0$, the total dissipation of angular momentum in the interval t_c is given by

$$\Delta \mathfrak{M} = \mathfrak{M}_0 \left(1 - x_f^{1/2}\right) \tag{18}$$

which, together with equation (17), gives a relation between $\Delta \mathfrak{M}$ and τ_1. If we now combine equation (18) with equation (13), the following relation is obtained between mass ~~dissipate~~ and angular momentum dissipation:

$$\frac{\Delta \mathfrak{M}}{\mathfrak{M}_0} = 1 - \left(1 - \frac{4 m \ell^2}{\mu a_0^2}\right)^{1/4} \tag{19}$$

In order to reduce $\mathfrak{M}$ by an appreciable amount, $4m\ell^2/\mu a_0^2$ must be of the order of one. If $m/\mu \ll 1$ as has been assumed, ℓ/a_0 must be much greater than one. Hence the critical point of the present theory is the value of ℓ. If we assume that during magnetic activities in the early phase of evolution the magnetic field prevailing on the stellar surface is of the order of 10^4 gauss, and as a dipole field it decreases with the distance, d, according to d^{-3}, then at a distance of $10^3 R$ the stellar field will reach the same strength as that of the interstellar magnetic field, say 10^{-5} gauss (Chandrasekhar and Fermi 1953). Therefore $10^3 R$ may be the upper limit of ℓ. Hence, if $a_0 > 10^3 R$, the proposed mechanism of braking orbital motion is not expected to be effective. Most likely only binaries with $a_0 < 10^2 R$ can be brought into contact by magnetic braking. On the other hand, we should remember that in the early stage of evolution the stellar radius R is large. This fact enhances the effectiveness of the suggested mechanism for converting close binaries into contact ones.

Case 2. Since we no longer assume contraction of component stars, the luminosity of both components become constant. $I_1 \leftrightarrow I_2$, denoted by I, can no longer be neglected if the two components are close. We have as the equation for angular momentum, assuming synchronization,

$$(I + \mu a^2)\frac{d\omega}{dt} + 2\mu a \omega \frac{da}{dt} = -\beta \ell^2 \omega, \tag{20}$$

if the total mass ejected is very small compared with the stellar masses. Integration of equation (20) yields

$$1 - x^2 - \frac{6I}{\mu a_0^2} \ln \frac{1}{x} = \frac{4t}{\tau_1} \tag{21}$$

where x and τ_1 are given by equation (12). Hence the time scale that the binary will become a contact one is of the order of $\tau_1/4$. The separation, x, of the two components decreases from 1, at first like $(1 - 4t/\tau_1)^{\frac{1}{2}}$, and then more rapidly when the logarithmic term becomes appreciable.

3. Evolution of Contact Binaries

Equations (14) and (21) are valid only before the two stars come into contact. If after the contact the angular momentum continues its dissipation, the binary orbit cannot further shrink without violating the stellar structure because a high pressure develops at the surface of contact. Actually the binary will follow a course that meets the least resistance as well as satisfies the condition of decreasing angular momentum. One can easily see that the course is to remove the mass in the surface layer of the less massive component to that of its companion. This may be regarded as a fusion phase of evolution of contact binaries.

Let us assume the two components to be main-sequence stars so that they satisfy the mass-radius relation,

$$\frac{R_i}{R_\odot} = \left(\frac{M_i}{M_\odot}\right)^{0.7}, \tag{22}$$

obtained by Russell and Moore (1940). Hence the separation between the components at any time becomes

$$a = R_1 + R_2 = R_\odot \left[\left(\frac{M_1}{M_\odot}\right)^{0.7} + \left(\frac{M_2}{M_\odot}\right)^{0.7} \right]. \tag{23}$$

As the angular momentum of the contact binary system decreases, mass flows from M_2 to M_1 if $M_1 > M_2$. In other words, $dM_2/dt = -dM_1/dt < 0$, as the total mass of the system is assumed to be not perceptibly lowered by the ejection process. Equation of angular momentum dissipation then becomes

$$\frac{dI}{dt} + 2\mu a \frac{da}{dt} + a^2 \frac{d\mu}{dt} + (I + \mu a^2) \frac{1}{\omega} \frac{d\omega}{dt} = -\beta \ell^2 \tag{24}$$

With the aid of equations (2) - (3) and (22-23), equation (24) can be reduced to

$$\left(\frac{M}{M_\odot}\right)^{2.4} f(\xi) \frac{d\xi}{dt} = \frac{1}{\tau_2} \tag{25}$$

where

$$\tau_2 = \frac{M_\odot R_\odot^2}{\mu l^2} \tag{26}$$

and

$$\begin{aligned} f(\xi) = {} & 2.4\,k^2(\xi^{1.4} - \eta^{1.4}) - 0.35\,\xi\eta(\xi^{0.7} + \eta^{0.7})(\eta^{-0.3} - \xi^{-0.3}) \\ & - (\xi - \eta)(\xi^{0.7} + \eta^{0.7})^2 \\ & + 1.05\,k^2 \frac{(\xi^{2.4} + \eta^{2.4})(\eta^{-0.3} - \xi^{-0.3})}{\xi^{0.7} + \eta^{0.7}} \end{aligned} \tag{27}$$

and

$$M_1 = \xi M, \quad M_2 = \eta M, \quad M_1 + M_2 = M \tag{28}$$

with

$$k_1 = k_2 = k.$$

Integration of equation (25) gives

$$\left(\frac{M}{M_\odot}\right)^{2.4}\left[\varphi(\xi) - \varphi(\xi_0)\right] = \frac{t}{\tau_2} \tag{29}$$

where

$$\varphi(\xi) = \int_{0.5}^{\xi} f(\xi)\, d\xi \tag{30}$$

and ξ_0 is the initial value of ξ .

Since it is always in the direction from the less to the more massive component that the mass flows in the course of angular momentum dissipation,

$\xi \geq 0.5$ always. This explains the lower limit adopted in the integral defining $\varphi(\xi)$.

Table 1 gives functions $\varphi(\xi)$ as defined by equation (30) as well as $\psi(\xi)$

$$\psi(\xi) = \xi^{0.7} + \eta^{0.7}, \tag{31}$$

which is related to the separation of the two components by

$$a = R_{\odot} \left(\frac{M}{M_{\odot}}\right)^{0.7} \psi(\xi). \tag{32}$$

The case of $\xi = 1$ corresponds to the disappearance of the less massive component. Ironically by annexing its companion the more massive component is overtaken by its own instability, because it can be easily seen that at the moment of complete merging of the two components the resultant star is rotationally unstable. However, the rotationally unstable star has less angular momentum than the preceeding state of being a contact binary with a large disparity in mass.

The total time from the first contact to the complete disappearance of the less massive component is equal to

$$t = \left(\frac{M}{M_{\odot}}\right)^{2.4} \tau_2 \left[\varphi(1) - \varphi(\xi_0)\right]. \tag{33}$$

For two stars with equal masses in the beginning, $\xi_0 = 0.5$,

$$t = 0.27 \left(\frac{M}{M_{\odot}}\right)^{2.4} \tau_2 \tag{34}$$

from Table 1. Hence τ increases with $M^{2.4}$ provided that τ_2 does not vary with M. Actually if the ejection of mass is directly related to the luminosity, τ may decrease with mass because of possible higher rates of dissipation of angular momentum in stars of higher luminosities. Hence a more definite statement can be made only after we have understood quantitatively the actual loss of angular momentum through mass ejection.

That the mass flows from the less to the more massive component in a contact binary during angular momentum dissipation is due to the mass-radius relation given by equation (23). This makes a decrease of separation, $a = R_1 + R_2$, correspond to a transfer of mass from the less to the more massive component.

4. Discussion

While the present suggestion for the formation of contact binaries is ideally sound because it is based entirely on known physical principles, and empirically supportable because we do find evidence of magnetic activities in the early phase of stellar evolution, we would like, in addition, to mention a few observational results which may be related to the present suggestion.

Mass must flow from the less to the more massive component in a contact configuration if the angular momentum of the system is being continuously dissipated. Hence unless the dissipation stops just when two stars of equal masses come into contact, the masses of two components in a contact binary will in general differ from each other. In other words, the chance of finding two component stars of equal masses in contact binaries must be very small. Indeed, if we now examine the mass ratio of W UMa stars, we find that although the two components have usually similar spectral types, their masses are never equal. In Sahade's (1962) recent compilation there are listed 15 W UMa systems with known masses for both components. Table 2 gives the distribution of the mass ratio M_1/M_2 of these 15 systems. If we further remember that binaries of equal masses are the easiest to be detected, the distribution as given in Table 2 shows clearly the avoidance of mass ratio around 1 by these binary systems. On the other hand, when we examine the mass ratios of non-contact binaries we find that binaries with components of equal masses are quite common. As examples we may cite YY Gem, RW UMa, and WZ Oph, all having a mass ratio of one, and DI Her, SS Boo, and CV Vel all having mass ratios nearly one. Thus the difference in the mass ratio found between contact binaries and non-contact binaries appears to favor our present suggestion. On the other hand, it may be due to pure chance.

A certain inverse relation exists between the frequency occurrence of spectroscopic binaries and the state of axial rotation in different groups of stars. Smith and Struve (1944) concluded from a study of 71 Pleiades stars that there was a marked scarcity of large-amplitude binaries in that cluster. On the other hand, the average rotational velocity of stars in the cluster was known to be above the average value derived from field stars of the same spectral types. This puzzling relation has been more clearly shown in a recent paper by Abt and Hunter (1962). They have not only confirmed Smith and Struve's result for the Pleiades stars but also found a reverse phenomenon, namely, those clusters whose stars are low in the observed rotational velocities contain high percentages of spectroscopic binaries. They have derived this conclusion from a study of three groups of stars — the I Lac and I Ori associations and the α Per cluster. Later, Abt and Snowden (1964) have further confirmed this result in the cluster IC 4665 from their study of radial velocities and Deutsch's (1955) study of rotational velocities. In the case of Orion stars, the same result has been obtained by McNamara and Larsson (1962) and McNamara (1963).

We have suggested the decrease of separations of non-contact binaries and the merging of two components in contact binaries into single rotational stars as a result of the loss of angular momentum. Hence, if the condition should be favorable to dissipation of angular momentum, such as the presence of a gaseous medium in the surroundings, we would expect many spectroscopic binaries to become rapidly rotating stars in this way. This may be the state in the Pleiades cluster. On the other hand, if the dissipation is unfavorable or the associations or clusters are so young that little dissipation has taken place, the percentage of spectroscopic binaries will maintain their original proportion. Such may be the case in the associations. However, we do not claim that this is indeed the cause for the negative correlation between populations of rapidly rotating stars and of spectroscopic binaries, because there are many other interpretations, such as Abt's (1965), for this fact. Parenthetically it may be said that whatever is the true cause, this negative correlation between two kinds of stellar objects both possessing large amounts of angular momentum points out most clearly that objects of large angular momentum should be viewed in an overall manner instead of being discussed separately in the name of binaries, rotating stars and planetary systems.

A consequence of the mass transfer proposed here for the merging of two components in a contact binary into a single rotating star is the fact that it turns the less massive component inside out, because according to our suggestion the loss of mass from the less massive component resembles the peeling of an onion. Consequently, if thermonuclear reactions of converting hydrogen into helium have started in the interior of the less massive component before this mode of mass transfer sets in, we will find a higher helium abundance in the atmosphere of the resulting stars than the amount present in the atmospheres of two original component stars. Hence it is interesting to note that there are stars such as upsilon Sagittarii (Greenstein 1940), HD124448 (Popper 1947), HD30353 (Bidelman 1950), HD160641 (Bidelman 1952), HD168476 (Thackeray 1954), and HD96446 (Jaschek and Jaschek 1959), which show unusually high abundance of helium and/or low abundance of hydrogen. However at the present stage it is difficult to say whether such stars have indeed any connection with the mass transfer mechanism proposed here.

A consequence of the present theory that is difficult to check will be seen when we inquire about the orbital angular momentum that has been dissipated in the course of orbital contraction. Previously we have suggested (Huang 1965ab) that the angular momentum that is lost by rotating stars through a similar braking process may have gone to the surrounding medium that is remnant of star formation. We have further speculated that this medium, after having acquired angular momentum, will collapse into a disk structure from which a planetary system may emerge. What is said of braking rotating stars obviously applies also to the braking of orbital motion. Consequently we would expect that planetary systems around W UMa stars might be common. Since the orbital angular momenta of binaries are much greater than those of rapidly spinning stars, we may also conjecture that the planets, if indeed formed at all, will be much farther away from the parent binaries. On the other hand, because of the large angular momentum, the surrounding medium may become so dispersed that it never reaches densities high enough for condensation.

I would like to express my sincere thanks to Mr. Clarence Wade, Jr. for the evaluation of the various functions and integrals that appear in this paper on the IBM 7094 computer at Goddard. This investigation was supported in part by the National Aeronautics and Space Administration under grant number NGR 14-007-041.

BIBLIOGRAPHY

Abt, H. A. 1965, Ap. J. Suppl. 11, 429.

Abt, H.A. and Hunter, J.H., Jr. 1962, Ap. J., 136, 381.

Abt, H.A. and Snowden, M. S., 1964, Ap. J., 139, 1139.

Babcock, H. W. and Babcock, H. D. 1958, "Electromagnetic Phenomena in Cosmical Physics (I.A.U. Symposium No. 6)", ed. B. Lehnert (Cambridge: Cambridge University Press) p. 239.

Bidelman, W. P. 1950, Ap. J. 111, 333.

Bidelman, W. P. 1952, Ap. J., 116, 227.

Bonsack, W. K. and Greenstein, J. L. Ap. J., 131, 83.

Chandrasekhar, S. and Fermi, E. 1953, Ap. J., 118, 113.

Deutsch, A. J. 1955, Principes fondamentaux de classification stellaire (colloq. Internat. Centre Natl. Rech. Sci. Paris 1953) p. 25.

Fowler, W. A., Greenstein, J. L. and Hoyle, F. 1962, Geophys. J. R.A.S. 6, 148.

Greenstein, J. L. 1940, Ap. J., 91, 438.

Hayashi, C. 1961, Pub. Astr. Soc. Japan, 13, 450.

Herbig, G. H. 1957, "Non-Stable Stars", (Cambridge: Cambridge University Press) p. 3.

Herbig, G. H. 1962, Symposium on Stellar Evolution, (La Plata: Astr. Obs. Nat. U. LaPlata) p. 23.

Huang, S.-S. 1965a, Ap. J., 141, 985.

Huang, S.-S. 1965b, Pub. A.S.P., 77, 42.

Hynek, J. A., 1951, Astrophysics, (New York: McGraw-Hill Book Co.) Chap. 10.

Jaschek, M. and Jaschek, C. 1959, Pub. A.S.P. 71, 465.

Jeans, J. H. 1944, "The Universe Around Us" 4th ed. (Cambridge: Cambridge University Press) p. 226.

McNamara, D. H. 1963, Ap. J. 137, 316.

McNamara, D. H. and Larsson, H. J. 1962, Ap. J., 135, 748.

Osterbrock, D. E. 1961, Ap. J., 134, 347.

Popper, D. M. 1947, Pub. A.S.P., 59, 320.

Poveda, A. 1964, Nature, 202, 1319.

Roxbough, I. W. 1965, A. J. 70, 330.

Russell, H. N. and Moore, C. E. 1940, The Masses of the Stars, (Chicago: Univ. of Chicago Press).

Sahade, J. 1962, Symposium of Stellar Evolution, (La Plata: Astr. Obs. Nat. Univ. La Plata), p. 185.

Schatzman, E. 1962, Ann. d'Ap. 25, 18.

Shapley, H. 1948, Centennial Symposia (Harvard Obs. Monographs No. 7) p. 249.

Smith, B. and Struve, O. 1944, Ap. J., 100, 360.

Struve, O. 1950, "Stellar Evolution" (Princeton: Princeton University Press).

Thackeray, A. D. 1954, M. N., 114, 93.

Wilson, O. C. 1963, Ap. J. 138, 832.

TABLE 1

Functions Determining the Separation and Mass Transfer

Between Two Components of the Contact Binary

ξ	$\psi(\xi)$	$\varphi(\xi)$	ξ	$\psi(\xi)$	$\varphi(\xi)$
.50	1.2311	0	.69	1.2118	.04847
.51	1.2311	.00014	.70	1.2096	.05358
.52	1.2309	.00055	.71	1.2072	.05891
.53	1.2307	.00123	.72	1.2048	.06448
.54	1.2303	.00219	.73	1.2022	.07027
.55	1.2298	.00343	.74	1.1994	.07628
.56	1.2293	.00493	.75	1.1965	.08250
.57	1.2286	.00671	.76	1.1935	.08892
.58	1.2278	.00875	.77	1.1903	.09555
.59	1.2269	.01106	.78	1.1869	.10237
.60	1.2259	.01364	.79	1.1833	.10938
.61	1.2248	.01649	.80	1.1795	.11656
.62	1.2236	.01960	.81	1.1756	.12391
.63	1.2223	.02296	.82	1.1714	.13143
.64	1.2208	.02659	.83	1.1670	.13909
.65	1.2192	.03047	.84	1.1624	.14690
.66	1.2176	.03460	.85	1.1575	.15484
.67	1.2157	.03898	.86	1.1523	.16289
.68	1.2138	.04361	.87	1.1469	.17105

ξ	$\psi(\xi)$	$\varphi(\xi)$
.88	1.1411	.17929
.89	1.1350	.18761
.90	1.1284	.19598
.91	1.1215	.20438
.92	1.1140	.21279
.93	1.1059	.22117
.94	1.0972	.22951
.95	1.0876	.23774
.96	1.0769	.24581
.97	1.0648	.25364
.98	1.0506	.26110
.99	1.0328	.26791
1.00	1.0000	.27273